Electromagnetic Radiation and Scattering of Wire Structures

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Abstract

Electromagnetic radiation and scattering are considered important topics in electrical engineering and technology, and help form the foundation of electromagnetic interference and electromagnetic compatibility. A comprehensive understanding of the radiation and scattering phenomenon will benefit the students who want to be engineers in the fields of RF circuit design, antennas and wireless communications, Radar engineering, and high-speed networking. This paper will discuss the physics of electromagnetic radiation and scattering of wire structures, which can be adopted to understand more complicated applications. After an introduction of the basic theory, discussion of the phenomena from the perspective of engineering is presented. Numerical simulations from time domain will be provided for several fundamental wire structures. This effort fosters the students’ interest in electrical engineering. The University of Southern Mississippi is in the process of enhancing its electronic engineering technology program, and materials presented in this paper will be utilized in the development of future courses.

1 Introduction

Frequency and time domain methods are two basic categories in computational electromagnetics. The frequency domain methods (FD) [1] deal with one frequency at a time and are thus usually used in harmonic analysis and are efficient for narrow-band applications. In time domain methods (TD), the time is discretized and thus usually a Marching-on-in-time (MOT) scheme [2] can be obtained. There is no explicit frequency variable involved in time domain methods. A single analysis in time gives broadband results, thus TD methods are efficient for wide band simulations. By making use of the Fourier transform and the inverse Fourier transform, properties in one domain can be obtained from results obtained from another domain.

Another property of the time domain methods is that the algorithms can be terminated easily in time, and can be used to give us solutions for as long as we want. This is useful in transient applications where sometimes we care only the early time response.

Thin wire structures are simple, but they can be considered component of more complicated structures. Electromagnetic radiation and scattering of wire structures can be adopted to understand more complicated applications. Here, we choose straight thin wire scatterers as examples. The induced current and surface charges calculated by time domain method are able to help us visualize what really happen.

2 Formulation

Maxwell’s equations [3] are the foundation of computational electromagnetics. Along continuity equation and boundary conditions, they are used to solve electromagnetic problems. Assuming $\varepsilon$ and $\mu$ are constants
in the constitutive relations, the time domain Maxwell’s equations in the differential form are

\[ \nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}, \]  
(1)

\[ \nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} + \mathbf{J}^i(\mathbf{r}, t), \]  
(2)

\[ \nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{q_i(\mathbf{r}, t)}{\varepsilon}, \]  
(3)

\[ \nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0. \]  
(4)

where \( \mathbf{E}(\mathbf{r}, t) \) is the electric field intensity; \( \mathbf{H}(\mathbf{r}, t) \) is the magnetic field intensity; \( \varepsilon \) is the permittivity of the medium; \( \mu \) is the permeability of the medium; \( \mathbf{J}^i(\mathbf{r}, t) \) is the induced surface current density; \( q_i(\mathbf{r}, t) \) is the induced volume charge density; \( \mathbf{r} \) is the position vector and \( t \) is the time variable.

Define vector potential \( \mathbf{A}(\mathbf{r}, t) \) that is computed from the induced current

\[ \mathbf{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_{\omega} \frac{\mathbf{J}^i(\mathbf{r}', t')}{R} dv', \]  
(5)

and scalar potential \( \Phi(\mathbf{r}, t) \) that is computed from the induced charges

\[ \Phi(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon} \int_{\omega} \frac{q_i(\mathbf{r}', t')}{R} dv'. \]  
(6)

where the retarded time \( t' = t - t^d \) and \( t^d \) is the time for the wave to travel the distance between the observation and the source points. We have

\[ \mathbf{E}(\mathbf{r}, t) + \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} = -\nabla \Phi(\mathbf{r}, t), \]  
(7)

On the surface of the (assumed) perfect electrically conducting structure, the EM boundary condition requires that

\[ [\mathbf{E}^i(\mathbf{r}, t) + \mathbf{E}(\mathbf{r}, t)]_t = 0, \]  
(8)

where \( \mathbf{E}^i(\mathbf{r}, t) \) is the incident field and subscript \( t \) stands for tangential component. Then

\[ \left[ \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} + \nabla \Phi(\mathbf{r}, t) \right]_t = [\mathbf{E}^i(\mathbf{r}, t)]_t. \]  
(9)

Equation 9 is the time domain electric field integral equation (TD EFIE) used to analyze general scattering problems.

In the case of wire scatterers, we will replace \( \mathbf{J}^i(\mathbf{r}', t) \) and \( q_i(\mathbf{r}', t) \) with line current \( \mathbf{I}(\mathbf{r}', t) \) and linear charge density \( q_l(\mathbf{r}', t) \). For thin wire scatterers, the linear charge density \( q_l(\mathbf{r}, t) \) is related to the induced line current \( \mathbf{I}(\mathbf{r}, t) \) by continuity equation

\[ \frac{\partial q_l(\mathbf{r}, t)}{\partial t} = -\frac{\partial \mathbf{I}(\mathbf{r}, t)}{\partial l}, \]  
(10)

where \( l \) is the parameter along the length of the wire scatterer.

We assume the “thin wire” approximation that the current only has components along the axis of the wire and the current flows only on the surface of the wire. There is no current, and thus conductivity, across...
the wire or around the circumference. The axial directed current can only change its value with distance along the wire. Even under this assumption, the finite radius of the wire scatterer must be considered when computing the scattered electromagnetic field induced by the currents. Although the current is assumed on the surface, if it is distributed uniformly around the wire, the radiated electromagnetic field is calculated by its equivalent current element located on the axis of the wire, and the testing is performed on the surface of the wire, as shown in Figure 1. Under “thin wire” assumption, we have only the \( z \) component for the vector potential,

\[
A^z(x,y,z,t) = \frac{\mu}{4\pi} \int_{z'=-h}^{h} \frac{I(z',t')}{R} \, dz',
\]

where \( R \) is the distance between the source point \((0,0,z')\) and observation point \(P(x,y,z)\). With only \( z \) component of \( \mathbf{A}(\mathbf{r},t) \), we have TDIE equation as

\[
\frac{\partial^2 A^z}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 A^z}{\partial t^2} = -\frac{1}{c^2} \frac{\partial E^z}{\partial t}, \quad \text{for } z \in (-h,h).
\]

Because the equivalent current is assumed to be on the axis and the testing is done on the surface of the scatterer, the retarded time is

\[
t^r = t - \frac{|z-z'|}{c}.
\]

The distance \( R \) in the denominator of Equation 11 is

\[
R = \sqrt{|z-z'|^2 + a^2}.
\]

The “thin wire” approximation allows us to avoid the singularity of the Green’s function at \( R = 0 \).

Approximating the derivatives in Equation 12 by central finite differences [4]:

\[
\frac{\partial^2 A^z}{\partial z^2} = \frac{A^z_{m+1,n-1} - 2A^z_{m,n-1} + A^z_{m-1,n-1}}{\Delta z^2}
\]

and

\[
\frac{\partial^2 A^z}{\partial t^2} = \frac{A^z_{m,n} - 2A^z_{m,n-1} + A^z_{m,n-2}}{\Delta t^2}
\]

we have

\[
A^z_{m,n} = 2A^z_{m,n-1} - A^z_{m,n-2} + (c\Delta t)^2 F_{m,n} + \left(\frac{c\Delta t}{\Delta z}\right)^2 \left[ A^z_{m+1,n-1} - 2A^z_{m,n-1} + A^z_{m-1,n-1} \right],
\]

where

\[
F_{m,n} = \frac{1}{c^2} \frac{\partial E^z}{\partial t}(zm,t_n).
\]

The vector potentials \( A^z_{m,n} \) are computed from the currents up to the present time step \( n \). Note that the currents at the end segments are always set to be zero for thin wire scatterers to enforce the boundary condition at the ends of the wire.

By choosing \( \frac{c\Delta t}{\Delta z} \leq 1 \), we obtain an explicit MOT scheme, which means that when we compute the present vector potential \( A^z_{m,n} \), the only unknown current involved is \( I_{m,n} \). All currents on other segments are known in a retarded time. By choosing \( \frac{c\Delta t}{\Delta z} = 1 \), the scheme is simplified to

\[
A^z_{m,n} = A^z_{m+1,n-1} + A^z_{m-1,n-1} - A^z_{m,n-2} + (c\Delta t)^2 F_{m,n-1},
\]

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The induced current is decomposed by pulse basis functions

\[ I(z,t) = \sum_{k=1}^{n_s} I_k(t)P_k(z), \]  

(20)

where \( P_k(z) \) are pulse basis functions. \( I_k(t) \) is the coefficient of the basis functions to be determined at each time step. Then

\[ A_{m,n}^{z} = \sum_{k=1}^{n_s} \kappa_{m,k} I_k \left( t_n - \frac{|z_m - z_k|}{c} \right), \]  

(21)

where \( n_s \) is the number of segments, \( \kappa_{m,k} \) is the contribution of current \( I_k \) on the \( k^{th} \) segment to the testing point \( m \).

\[ \kappa_{m,k} = \frac{\mu}{4\pi} \int_{e_l^k}^{e_r^k} \frac{1}{\sqrt{(|z_m - z'|^2 + \alpha^2)}} dz', \]  

(22)

\( e_l^k \) and \( e_r^k \) stand for the left and right edges of segment \( k \), respectively.

In the case of explicit MOT schemes, writing

\[ A_{m,n}^{z} = \kappa_{m,m} I_{m,n} + A_{m,n}^{z,H}, \]  

(23)

where \( A_{m,n}^{z,H} \) is the vector potential computed by the currents excluding those at the present time step \( n \):

\[ A_{m,n}^{z,H} = \sum_{k=1, \kappa \neq m}^{n_s} \kappa_{m,k} I_k \left( t_n - \frac{|z_m - z_k|}{c} \right). \]  

(24)

We have

\[ \kappa_{m,m} I_{m,n} = -A_{m,n}^{z,H} + 2A_{m,n-1}^{z} - A_{m,n-2}^{z} + (c\Delta t)^2 F_{m,n} \]

+ \( \left( \frac{c\Delta t}{\Delta z} \right)^2 \left( A_{m+1,n-1}^{z} - 2A_{m,n-1}^{z} + A_{m-1,n-1}^{z} \right) \) for each \( m \),

(25)

or

\[ \kappa_{m,m} I_{m,n} = -A_{m,n}^{z,H} - A_{m,n-2}^{z} + (c\Delta t)^2 F_{m,n} + A_{m+1,n-1}^{z} + A_{m-1,n-1}^{z}, \]  

(26)

for each \( m \), in the case of \( \frac{c\Delta t}{\Delta z} = 1 \).

3 Numerical Examples [5]

A perfectly conducting straight thin wire scatterer of radius \( a = 1 \) centimeter lies in the \( \hat{a} \) direction, as in Figure 1. It is illuminated by a Gaussian-pulsed plane wave trans-magnetic (TM) electric field

\[ \mathbf{E}^i(r,t) = E_0 \frac{4}{T \sqrt{\pi}} e^{-\frac{\gamma^2}{4}}, \]  

where \( \gamma = \frac{4}{T} (ct - ct_0 - \mathbf{r} \cdot \mathbf{a}^k) \),

(27)

where \( c \) is the velocity of propagation in the surrounding medium. The pulse width of the Gaussian impulse \( T = 6.67 \) nanoseconds. We are to compute the transient current distribution induced on the wire. This distribution gives us all the information needed for further computation of the scattering characteristics.
3.1 Short Wire Scatterer

The first example is a short wire scatterer with a length of 2 meters as in Figure 1. The polarization of the incident electric field is parallel with the wire axis. The wave propagates in the $+y$ direction. \( i.e., \)

\[
E_0 = 120\pi\hat{a}_z \quad \text{and} \quad \hat{a}_y = \hat{a}_z.
\]  

From Figure 2, we see that at the early stage of the response, the induced current is large because of the input field, while at later time, the induced current attenuates as the energy radiates into the space. In the early time, the induced current follows mostly the temporal characteristic of the input Gaussian pulse. As
the input field ceases, the inherent resonant property of the wire antenna dominates. This is plotted in figure 4.

Figure 4: Waveforms of the induced current on the wire

To further show the scattering phenomenon, we provide results for a short wire scatterer to which the polarization of the electrical field is inclined with 45 degrees, i.e.,

$$E_0 = 120\pi(-\hat{a}_x \sin \frac{\pi}{4} + \hat{a}_z \cos \frac{\pi}{4}), \text{ and } \hat{a}_k = \hat{a}_x \cos \frac{\pi}{4} + \hat{a}_z \sin \frac{\pi}{4}.$$ \hspace{1cm} (29)

The transient current with time at the center of the wire is plotted in Figure 5. The induced surface charges at one end of the wire are also plotted in Figure 6. From Figure 5, we see that as the direction of propagation is oblique to the wire axis, the response on the wire is a moving pulse back and forth between the ends A and B of the wire. At time $t_1$, the plane wave first illuminates end A of the wire and gives rise to a large induced current here. With time advances to $t_2$, the peak of the response arrives at end B and bounces back. The induced current at end B is larger than that at end A. At a late time $t_3$, the induced current is small all over the length of the wire.
The distribution of induced surface charges in the normal (TM mode) incidence is antisymmetrical with respect to the center, thus there are no surface charges at the center of the wire (not plotted here). This is not the case for slant incidence. The surface charges at the two ends are different. They are plotted in Figure 6 and Figure 7. The charge at the center, which is no longer always zero, is plotted in Figure 8.

![Figure 7: Induced surface charges at the other end of the short wire (slant incidence)](image)

![Figure 8: Induced surface charges at the center of the short wire (slant incidence)](image)

### 3.2 Long Wire Scatterer

Another example is a long wire of 10 meters and the incident electric field

\[
\vec{a} = \vec{a}_x \quad \text{and} \quad E_0 = 120\pi \vec{a}_z.
\]  

(30)

The length of the wire in this example is long compared with the pulse width of the incident field. The transient current induced in the center of the wire is plotted in Figure 9 and the surface charge at one end is plotted in Figure 10.

![Figure 9: Transient current at the center of the long wire (normal incidence)](image)

![Figure 10: Induced surface charges at one end of the long wire (normal incidence)](image)
From the simulation of short straight wire scatterer with normal incident field in Figure 2 and the simulation of long straight wire scatterer with normal incident field in Figure 9, we find that for the same duration of simulation time (1.333µs), the current response is quite different. One physical explanation is that the energy radiates faster into space for a short wire than for a long wire, therefore, the induced current is much smaller on the short wire scatterer than on the long wire scatterer after the incident field has ceases for a period of time. The simulation of the short wire scatterer can be terminated earlier than that of the long wire scatterer. That is why time domain methods are usually adopted in transient simulations. The spatial distribution of the induced current is obviously different with that for a short wire scatterer. In Figures 11, the induced current pulse has width in the order of the structure, while in Figure 12, the induced current pulse has width that is small compared with the structure. This is a feature of transient response on electrically large structure, on which the high order details are clustered in the vicinity of the prominent response.

4 Conclusions

In this paper, the scattering phenomena of straight thin wire structures is investigated. After introduction of the basic theory, the induced current and surface charges are calculated from time domain electric field integral equations. The time-domain results help visualize the activity of induced surface charges. The discussion of the physics of electromagnetic radiation and scattering of wire structures will help undergraduate students in the electronics engineering technology to understand more complicated applications.

References


Biography

Zhaoxian Zhou received the B. Eng. from the University of Science and Technology of China in 1991; M. Eng. from the National University of Singapore in 1999 and the PhD degree from the University of New Mexico in 2005. All degrees are in Electrical Engineering. From 1991 to 1997, he was an Electrical Engineer in China Research Institute of Radiowave Propagation. In the fall of 2005, he joined the School of Computing, the University of Southern Mississippi as an assistant professor. His research interests include Time Domain Computational Electromagnetics, High Performance Computing and Numerical Analysis.

Professor Randy Buchanan is the Director of the Instrumentation and Cryogenics Laboratory and Assistant Director for the School of Computing at the University of Southern Mississippi. His areas of research and interests include control systems, cryogenics, scientific instrumentation, laboratory automation, space hardware processing, and planetary & space simulation vacuum chamber systems.