A MathCAD Function Set for Solving Thermodynamics Problems

Stephen T. McClain

Abstract – A set of MathCAD functions was constructed to evaluate the thermodynamic properties of steam, R-134a, air, and twelve ideal gases. The functions were created to ease the need for time-consuming interpolation using tabularized thermodynamic data while reinforcing the functional representation of traditional thermodynamic property tables. A consistent and easily remembered scheme has been adopted for naming the functions. An example is provided to demonstrate the use of the functions in an undergraduate applied thermodynamics course. The example seeks the reheat pressure that maximizes the efficiency of a Rankine-reheat power cycle provided fixed boiler and condenser pressures. The example is presented in its entirety in the appendix to demonstrate the report-quality worksheets possible using MathCAD. While the example provided represents a use of the functions in an undergraduate applied thermodynamics course, the functions have also been used in an internal combustion engines course and in a turbomachinery course. Survey results from three courses and several student comments are discussed.

Keywords: Thermodynamics, MathCAD, steam properties, R-134a properties

INTRODUCTION

Learning with a combination of a textbook and a software package is a contemporary engineering-thermodynamics pedagogy. Many software tools are available for evaluating thermodynamic properties of engineering fluids. Many of these software tools are proprietary packages sold by textbook publishers, such as “Interactive Thermodynamics: IT” [1]. In fact, finding a thermodynamics text that does not come with a software package is difficult. Some textbooks are now built around using a software or web-based internet package [2]. While many educational software packages are available for evaluating thermodynamic properties, evidence showing that practicing engineers continue to use these thermodynamic-property software packages after entering the workforce is not readily available.

MathCAD, MatLab, and Engineering Equation Solver (EES) are all powerful computational and analytical packages [3,4,5]. Many schools teach and require the use of a computational tool such as MathCAD, MatLab, or EES [6]. From informal conversations with engineers who learned to use one of these computational tools, many of them continue to use these tools after graduation. Developing extensions or toolkits for software that the students will use after graduation seems more appropriate than developing complete software packages that will only be used by students in an educational environment. Because of the need for thermodynamic property functions for the widely used computational tools, functions were generated to evaluate the thermodynamic properties of water, R-134a, air, and twelve species of the CHON system in MathCAD. Equilibrium functions for eight independent reactions involving species of the CHON system are also included. The function set created provides most thermodynamic functions needed to teach a two-course sequence in undergraduate engineering thermodynamics.

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While developed for a two-course sequence in undergraduate thermodynamics, the function sets are useful for many other engineering courses. In fact, this effort began in an ME 448/548 Internal Combustion Engines course [7,8]. Undergraduates in thermodynamics based courses often become frustrated using property tables and fail to comprehend either the material or the significance of the material. As the case with all thermodynamic property evaluation software, the intent was 1) to take some of the effort and distraction away from working problems requiring extensive interpolation in the tables and 2) to allow the students to analyze more complicated problems.

Each computational analysis package has strengths and weaknesses when compared to the others. EES also has the thermodynamic functions discussed here (and functions for many other fluids). While EES is not a traditional structured programming language, the appearance of EES programs is similar to the appearance of C or FORTRAN programs, and some users find the unit conversion procedures awkward in EES. MathCAD was chosen for this project because of its mathematical report appearance, because of its ability to perform calculations with automatic unit handling and conversion, and because of its wide use in the Department of Mechanical Engineering at the University of Alabama at Birmingham.

REFERENCING THE FUNCTION SETS IN MathCAD

The functions are contained in three MathCAD worksheets. Table 1 presents the files and the information contained in each file. To use the functions in a new MathCAD worksheet, the information in the property worksheets does not have to be copied into the new worksheet. The function worksheet may be referenced by using the <Insert, Reference> command, and identifying the file required. When this is done correctly, a statement similar to

```mathcad
Reference:C:\ThermoII\SteamProps.mcd
```

will appear in the worksheet. All functions generated in referenced file will then be available for use in the new worksheet. The following three sections describe the individual functions available for MathCAD.

All of the functions generated are available to the educational public on the condition that if the functions are used for instruction, the instructor agrees to have his students complete a survey form at the end of the course. The function sets may be obtained by emailing the author at smcclain@uab.edu. Please include the subject line “MathCAD: Thermo Function Set Request” in your email.

Table 1. Property Function Files

<table>
<thead>
<tr>
<th>File</th>
<th>Information Contained</th>
</tr>
</thead>
<tbody>
<tr>
<td>SteamProps.mcd</td>
<td>Thermodynamic properties of water</td>
</tr>
<tr>
<td>R134aProps.mcd</td>
<td>Thermodynamic properties of R-134a</td>
</tr>
<tr>
<td>IGASProps.mcd</td>
<td>Thermodynamic ideal gas properties of CO, CO₂, H₂, H, OH, H₂O, N₂, N, NO, NO₂, O, O₂, and dry air and equilibrium constants for eight independent reactions</td>
</tr>
</tbody>
</table>

WATER AND R-134A

A consistent naming scheme was chosen for the functions evaluating properties of the phase change fluids. The function calls for water and R-134a begin with the variable to be determined, followed by an underscore, followed by the independent property(s), followed by a fluid identifier (H2O or R134a), followed finally by the values of the independent property(s) in parenthesis. Figure 1 presents the format of function calls for the phase change substances. The functions require that all independent properties be entered with the appropriate absolute units.

The functions were constructed to reflect and reinforce how property information is provided in traditional property tables. Functions are provided for the saturation pressure based on temperature and for the saturation temperature based on pressure. Saturated liquid and saturated vapor functions are provided for specific volume (ν), specific internal energy (u), specific enthalpy (h), and specific entropy (s) based on either temperature or pressure. Functions for ν, u, h, and s of saturated mixtures are also provided as functions of quality and either temperature or
Functions that return quality are provided. Functions that return temperature or enthalpy as functions of pressure and entropy, often required in the analyses of Rankine and reversed-Rankine cycles, are provided. Finally, a function that determines temperature based on pressure and enthalpy, often used when analyzing reversed-Rankine cycles, is provided. Table 2 presents a summary of the functions provided for the phase-change substances.

The property functions for water were created using the IAPWS 97 formulation [9]. The IAPWS formulation presents fundamental property equations for five regions. The functions provided are for Regions 1 (compressed liquid below 623.15 K), Region 2 (superheated vapor), and Region 4 (saturated mixture below 623.15 K). Functions for Region 3 (near and above the critical point) or Region 5 (low pressure very high temperature vapor) were not created but may be added at a later time.

Table 2. Functions for the Thermodynamic Properties of Water and R-134a

<table>
<thead>
<tr>
<th>Description</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns the saturation temperature as a function of pressure</td>
<td>Tsat_PXX(P)</td>
</tr>
<tr>
<td>Returns the saturation pressure as a functions of temperature</td>
<td>Psat_TXX(T)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for saturated liquid as a function of temperature</td>
<td>zf_TXX(T)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for saturated liquid as a function of pressure</td>
<td>zf_PXX(P)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for saturated vapor as a function of temperature</td>
<td>zg_TXX(T)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for saturated vapor as a function of pressure</td>
<td>zg_PXX(P)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for a saturated mixture as a function of temperature and quality</td>
<td>z_TXXX(T,x)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for a saturated mixture as a function of pressure and quality</td>
<td>z_PXXX(P,x)</td>
</tr>
<tr>
<td>Returns the property (v, u, h, or s) for either a compressed liquid or a superheated vapor as a function of temperature and pressure</td>
<td>z_TPXX(T,P)</td>
</tr>
<tr>
<td>Returns the quality of a saturated mixture as a function of the temperature and the property (v, u, h, or s)</td>
<td>x_TzXX(T,z)</td>
</tr>
<tr>
<td>Returns the quality of a saturated mixture as a function of the temperature and the property (v, u, h, or s)</td>
<td>x_PzXX(T,z)</td>
</tr>
<tr>
<td>Returns the temperature as a function of pressure and entropy</td>
<td>T_PsXX(P,s)</td>
</tr>
<tr>
<td>Returns the enthalpy as a function of pressure and entropy</td>
<td>h_PsXX(P,s)</td>
</tr>
<tr>
<td>Returns the temperature as a function of pressure and enthalpy</td>
<td>T_PhXX(P,h)</td>
</tr>
</tbody>
</table>

The reader should note that all functions presented in this paper are for educational use only, and that the user assumes all responsibility for calculations performed using the functions if used for an industrial application. For the steam functions in particular, the function results have been compared to steam table values; however, the functions have not been verified to the level required by the IAPWS 97 formulation for industrial use. If verified steam functions for MathCAD are required, Mathsoft no longer supports the steam functions that were once available in a MathCAD extension pack. Fortunately, steam functions from third-party software companies are available [10].

The property functions for R-134a are based on the equation of state proposed by Tillner-Roth and Baehr [11]. The functions are valid for temperatures from 170 K to 455 K at pressures up to 70 MPa. Because the functions are based on the formulation provided by Tillner-Roth and Baehr, energy and entropy values reported by the functions differ from the tabulated values reported in some undergraduate engineering thermodynamics texts that are based on the ASHRAE formulation [12].
AIR AND IDEAL GASES

The data used to create the functions for ideal gases and dry air came from “The Chemkin Thermodynamic Data Base” as reported by Turns [12]. Turns reports fourteen constants used to determine thermodynamic data for twelve species (CO, CO₂, H₂, H, OH, H₂O, N₂, N, NO, NO₂, O, O₂) of the carbon-hydrogen-oxygen-nitrogen (CHON) system as a function of temperature. The fourteen constants for each species are used to determine thermodynamic properties that are valid for the temperature range of 300 K to 5000 K. Table 3 summarizes the constants and functions available for the twelve ideal gases.

<table>
<thead>
<tr>
<th>Function or Constant</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mₔₓₓ</td>
<td>The molecular weight of the XX species.</td>
</tr>
<tr>
<td>Rₓₓ</td>
<td>The gas constant of the XX species.</td>
</tr>
<tr>
<td>hₘₓₓ(T)</td>
<td>Returns the molar specific enthalpy of the XX species as a function of temperature.</td>
</tr>
<tr>
<td>sₓₓ(T)</td>
<td>Returns the molar specific entropy of the XX species as a function of temperature.</td>
</tr>
<tr>
<td>uₓₓ(T)</td>
<td>Returns the molar specific internal energy of the XX species as a function of temperature.</td>
</tr>
<tr>
<td>μₓₓ(T)</td>
<td>Returns the molar specific Gibbs free energy of the XX species as a function of temperature.</td>
</tr>
<tr>
<td>cₚₓₓ(T)</td>
<td>Returns the molar specific heat at constant pressure of the XX species as a function of temperature.</td>
</tr>
<tr>
<td>cᵥₓₓ(T)</td>
<td>Returns the molar specific heat at constant volume of the XX species as a function of temperature.</td>
</tr>
</tbody>
</table>

The functions for ideal gases follow a slightly different naming scheme than used for the phase change substances. Since properties of ideal gases are functions only of temperature, the underscore and independent property identifiers found in the function names of the phase change substances are not present. An “m” follows the returned property variable in the ideal gas function names to remind the user that the functions return molar specific properties.

The ideal gas functions were used to generate the thermodynamic functions for air using a composition of 79% N₂ and 21% O₂. The specific internal energy, the specific enthalpy, and the standardized specific entropy were generated as functions of temperature. Reference values of the properties at 298 K were added so that the values reported by the functions equaled the values found in traditional ideal-gas air tables [13]. The functions were created on a gravimetric basis for the same reason. Along with the functions for internal energy, enthalpy, and entropy, functions were also created for the relative pressure and relative volume. The inverse functions, which find temperature from the other air properties, are also available in the worksheet. Table 4 presents the available ideal gas functions for air.

EQUILIBRIUM CONSTANTS

Chemical equilibrium functions are also included in the function set. Equilibrium constant functions were generated for eight independent reactions of the CHON system. Those eight reactions are:

I. \( H₂ \leftrightarrow 2H \)
II. \( O₂ \leftrightarrow 2O \)
III. \( N₂ \leftrightarrow 2N \)
IV. \( H₂ + \frac{1}{2}O₂ \leftrightarrow H₂O \)
V. \( 2H₂O \leftrightarrow H₂ + 2OH \)
VI. \( N₂ + O₂ \leftrightarrow 2NO \)
VII. \( CO₂ \leftrightarrow CO + \frac{1}{2}O₂ \)
Table 4. Functions for the Thermodynamic Properties of Air

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{\text{Tair}}(T) )</td>
<td>Returns the enthalpy of dry air as a function of temperature.</td>
</tr>
<tr>
<td>( u_{\text{Tair}}(T) )</td>
<td>Returns the internal energy of dry air as a function of temperature.</td>
</tr>
<tr>
<td>( s_{\text{Tair}}(T) )</td>
<td>Returns the standardized specific entropy of dry air for a reference pressure of 1 atm.</td>
</tr>
<tr>
<td>( \text{pr}_{\text{Tair}}(T) )</td>
<td>Returns the relative pressure of dry air as a function of temperature.</td>
</tr>
<tr>
<td>( \text{vr}_{\text{Tair}}(T) )</td>
<td>Returns the relative volume of dry air as a function of temperature.</td>
</tr>
<tr>
<td>( T_{h_{\text{Tair}}}(h) )</td>
<td>Returns the temperature of dry air as a function of enthalpy.</td>
</tr>
<tr>
<td>( T_{u_{\text{Tair}}}(u) )</td>
<td>Returns the temperature of dry air as a function of internal energy.</td>
</tr>
<tr>
<td>( T_{s_{\text{Tair}}}(s) )</td>
<td>Returns the temperature of dry air as a function of standardized specific entropy.</td>
</tr>
<tr>
<td>( \text{pr}_{\text{Tair}}(T) )</td>
<td>Returns the temperature of dry air as a function of relative pressure.</td>
</tr>
<tr>
<td>( \text{vr}_{\text{Tair}}(T) )</td>
<td>Returns the temperature of dry air as a function of relative volume.</td>
</tr>
</tbody>
</table>

The functions for the equilibrium constant are called using “KpYY(T)”, where “YY” represents the roman numeral listed for each reaction. For example, the equilibrium constant for \( \text{CO}_2 + \text{H}_2 \leftrightarrow \text{CO} + \text{H}_2\text{O} \) at 4500 K is found in MathCAD using the statement:

\[ \text{Kp}_{\text{VIII}}(4500\text{K}) = 8.932 \]

The equilibrium constant functions were validated using data from the *JANAF Thermochemical Tables* as reported by Russell and Adebiyi [12].

**EXAMPLE**

An example problem is provided demonstrating the use of the function sets in the analysis of thermodynamic cycles. The example included relates to the optimization of a Rankine power cycle with reheat. The example demonstrates how the functions relieve the need for tedious or repetitive interpolation in the thermodynamic property tables. The full solution of the example problem is provided in the appendix to demonstrate how the functions are not just property evaluators. Because of the format of MathCAD, the functions are used to create report-quality worksheets that may be submitted by the students as homework solutions.

**Optimization of a Rankine Power Cycle with Reheat**

*Problem Statement:* A Rankine power cycle with reheat operates between a boiler pressure of 1000 psia, a maximum cycle temperature of 1260 °R, and a condenser pressure of 2 psia. Steam exits the reheat boiler at 1260 °R. By varying the reheat boiler pressure between 5 psia and 900 psia, determine the reheat boiler pressure that maximizes the cycle efficiency and the resulting maximum efficiency.

The complete MathCAD solution for this example is presented in the appendix. The solution to this example demonstrates the use of the property functions for steam and demonstrates how MathCAD is used to increment or “sweep” variables.

A reheat pressure exists between the main boiler pressure and condenser pressure that maximizes the cycle efficiency. The problem statement indicates the optimum reheat pressure is to be found by “sweeping” the reheat pressure between 5 psia and 900 psia. In MathCAD a variable with a range of values may be created using a

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counting variable and creating an array using the counting variable as the index. In the MathCAD solution, the variable “i” is defined as a variable ranging from 0 to N_d, where N_d is the number of increments desired for the sweep. The range of reheat pressure values is then constructed by assigning the reheat pressure, P_{rh}, a counting subscript and defining each pressure as a function of the counting variable. The range variable construction statement appears in the MathCAD solution as:

\[
\begin{align*}
  i & := 0 .. N_d \\
  P_{rh_i} & := 5 \text{ psi} + \frac{i}{N_d} (900 \text{ psi} - 5 \text{ psi})
\end{align*}
\]

The variable P_{rh} is a vector with N_d + 1 elements ranging from 5 psi to 900 psi. All of the state properties following the reheat boiler and to entering the pump, must also be evaluated as arrays using the counting variable as the index. The construction is used to easily determine the reheat boiler pressure that maximizes the cycle efficiency.

**PEDAGOGY AND DELIVERY**

The MathCAD functions were presented to students in three applied thermodynamics courses during the Spring 2005 semester. The three courses were: ME 242 – Thermodynamics II (at the University of Alabama at Birmingham), ME 448 – Internal Combustion Engines (at the University of Alabama at Birmingham), and MECH 4703 – Aerospace Propulsion (at John Brown University).

In ME 242 and ME 448, the students were given access to the function files and documentation. The students were also provided a one-hour, in-class MathCAD introduction. For each major topic in ME 242 and ME 448, a lecture would be provided on the theory, and examples would be provided using traditional methods of problem solution. Solutions to the same examples would then be provided using the MathCAD functions. In some cases in ME 242, more examples were worked in class using a laptop and computer projector system. The students were required to use the functions in selected homework problems and on their projects. The students were not allowed to use the MathCAD functions to take their tests. The students in MECH 4703 were provided the function files and documentation, but were not given an in-class MathCAD introduction.

**SURVEY RESULTS AND STUDENT COMMENTS**

A survey was developed to explore the effectiveness of teaching the courses using the MathCAD functions. The survey consisted of twelve questions to which the students could reply: strongly agree (5), agree (4), neutral (3), disagree (2), and strongly disagree (1). The survey questions and the student responses, by course and overall, are presented in Table 5.

While rigorous statistical analyses have not been performed on the survey results (nor metrics developed), the results in Table 5 provide several interesting observations. Most importantly, the overall student response to using the MathCAD functions was highly positive. The overall student response to Statements #8 and #10 demonstrate that the functions and MathCAD made completing their homework easier and that the students preferred to use MathCAD to complete their homework assignments. The responses to Statement #3, especially from ME 242, in which most students had no MathCAD experience, demonstrates that students can learn to use MathCAD and the thermodynamic functions in a very short period of time.

Two interesting observations may be made regarding the effectiveness of the functions related to course instructions. First, the student responses were “neutral” to “agree” regarding the helpfulness of the MathCAD functions in learning the course material (Statement #9). Second, a disparity seems to exist between the student responses to Statements #8 and #9 compared to the response to Statement #10, especially for ME 242, Thermodynamics. These two observations seem to imply that while the functions made solving their homework problems easier, some students preferred to solve the homework problems in a more difficult and time consuming manner and that the functions were only marginally effective in improving learning for some students.
Table 5. MathCAD Survey Results

<table>
<thead>
<tr>
<th>Statement about the use of MathCAD in this course.</th>
<th>ME 242</th>
<th>ME 455</th>
<th>MECH 4703</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I had significant programming experience (2 hours credit or equivalent) in C or FORTRAN before using MathCAD in this course.</td>
<td>4.118</td>
<td>2.778</td>
<td>4.571</td>
<td>3.848</td>
</tr>
<tr>
<td>2. I have explored or used IT: Interactive Thermodynamics or EES software in my basic thermodynamics course.</td>
<td>2.235</td>
<td>1.778</td>
<td>3.000</td>
<td>2.273</td>
</tr>
<tr>
<td>3. A MathCAD introduction was provided in this course, and the introduction was sufficient to get me started using MathCAD.</td>
<td>4.647</td>
<td>4.556</td>
<td>2.286</td>
<td>4.121</td>
</tr>
<tr>
<td>4. I completed a numerical analysis or programming course in my engineering curriculum prior to my enrollment in this course.</td>
<td>3.188</td>
<td>2.667</td>
<td>3.857</td>
<td>3.188</td>
</tr>
<tr>
<td>5. The documentation and examples provided regarding the MathCAD functions were understandable and helpful.</td>
<td>4.412</td>
<td>4.444</td>
<td>4.286</td>
<td>4.394</td>
</tr>
<tr>
<td>6. The automatic unit handling and conversion features of MathCAD were easy to understand and control.</td>
<td>4.176</td>
<td>4.111</td>
<td>4.714</td>
<td>4.273</td>
</tr>
<tr>
<td>7. I used the MathCAD functions for multiple homework solutions in this course.</td>
<td>4.294</td>
<td>4.667</td>
<td>5.000</td>
<td>4.545</td>
</tr>
<tr>
<td>8. I preferred to use the MathCAD functions for my homework solutions as opposed to using thermodynamic property tables.</td>
<td>3.882</td>
<td>4.000</td>
<td>4.857</td>
<td>4.121</td>
</tr>
<tr>
<td>9. The MathCAD functions helped me learn the material in this course.</td>
<td>3.588</td>
<td>3.556</td>
<td>3.571</td>
<td>3.576</td>
</tr>
<tr>
<td>10. The thermodynamic property functions in MathCAD made solving problems easier.</td>
<td>4.824</td>
<td>4.333</td>
<td>4.714</td>
<td>4.667</td>
</tr>
<tr>
<td>11. Because of my experience using MathCAD in this course, I will use MathCAD in subsequent engineering courses.</td>
<td>4.647</td>
<td>4.556</td>
<td>4.714</td>
<td>4.636</td>
</tr>
<tr>
<td>12. Because of my experience using MathCAD in this course, I will be more likely to use MathCAD in my engineering practice.</td>
<td>4.588</td>
<td>4.333</td>
<td>5.000</td>
<td>4.606</td>
</tr>
</tbody>
</table>

However, after post course interviews with the students and after reflection by the ME 242 professor, the two observations are most likely due to the facts that 1) some students became dependent on the MathCAD functions to solve their thermodynamics problems, 2) the students were not allowed to use their computers and the MathCAD functions during the course tests, and 3) the professor gave tests requiring manual solution techniques and failed to recognize that the pedagogical changes required changes in testing and assessment. Some of the students in the ME 242 course took advantage of the MathCAD resources in completing almost all of their homework assignments, yet the same students performed poorly on the tests which were created to evaluate the students ability to use the tabular property data. The student responses reflect the need to adapt testing methods and expectations as the course content and delivery changes.

**CONCLUSIONS**

A set of functions for evaluating the thermodynamic properties of many working fluids has been created for MathCAD. An example problem using the functions is presented in its entirety in the appendices to demonstrate the ability of MathCAD to create report quality solutions to applied thermodynamics problems.

The function sets have been used recently in three undergraduate, applied thermodynamics courses. The functions were highly successful in allowing the students to complete complicated homework and project assignments. The function sets eased the effort of completing the homework assignments that otherwise require the arduous use of tabularized thermodynamic property data. Student response to using the functions and MathCAD was also highly positive. While student response was positive, the survey and other feedback from the students identified the need to adjust testing methods when MathCAD and the thermodynamic functions are used heavily during instruction.
ACKNOWLEDGEMENTS

The author thanks Bharat K. Soni and the Department of Mechanical Engineering at the University of Alabama at Birmingham for their support of these teaching activities. The author would also like to thank his students for their enthusiasm, dedication (or perseverance in some cases), and professionalism. The author also thanks Dr. Kevin Macfarlan and his students at John Brown University for their use of the MathCAD functions, for their participation in the survey, and for their excellent feedback regarding the functions.

REFERENCES


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Stephen T. McClain is an Assistant Professor at the University of Alabama at Birmingham. He received his B.S. in mechanical engineering from The University of Memphis in 1995, and he received his M.S. (1997) and Ph.D. (2002) degrees in mechanical engineering from Mississippi State University. Dr. McClain has taught classes in thermodynamics, fluid mechanics, internal combustion engines, instrumentation, experimental design, and uncertainty analysis.
Appendix - Rankine-Reheat Power Cycle Optimization

Reference: C:\ThermoII\SteamProps.mcd

Problem Statement: A Rankine power cycle with reheat operates between a boiler pressure of 1000 psia and a condenser pressure of 2 psia. The maximum boiler temperature and reheat boiler temperature are both 1260 R. By varying the reheat boiler pressure between 5 psia and 900 psia, determine the reheat boiler pressure that maximizes the cycle efficiency and the resulting maximum efficiency.

Sketches: The schematic and T-s diagrams for a Rankine cycle with reheat follow.

Solution: The solution begins by entering the cycle parameters identified in the problem statement.

\[
\begin{align*}
P_{\text{boiler}} &= 1000 \text{ lbf/in}^2 \\
P_{\text{cond}} &= 2 \text{ lbf/in}^2 \\
T_{\text{bmax}} &= (860 + 460) \text{ R} \\
T_{\text{bmax}} &= 700 \text{ K}
\end{align*}
\]

To vary the reheat boiler pressure, a counting variable is identified ranging from 0 to 50. A vector of reheat pressures between 5 and 900 psia is then created using the counting variable.

\[
N_d := 50 \quad i := 0 \ldots N_d \quad P_{rh_i} := 5 \text{ psi} + \frac{i}{N_d} \cdot (900 \text{ psi} - 5 \text{ psi})
\]

The properties at each state are then evaluated starting at State 1: which is assumed to be a saturated liquid entering the pump. For State 1, the temperature, enthalpy, and specific volume are required.

\[
\begin{align*}
T_1 &= T_{\text{sat, H}_2\text{O}(P_{\text{cond}})} \\
T_1 &= 585.745 \text{ R} \\
h_1 &= h_{\text{f, H}_2\text{O}(P_{\text{cond}})} \\
h_1 &= 94.034 \text{ BTU/lb}
\end{align*}
\]

\[
\begin{align*}
v_1 &= v_{\text{f, H}_2\text{O}(P_{\text{cond}})} \\
v_1 &= 0.016 \text{ ft}^3/\text{lb}
\end{align*}
\]

The enthalpy at State 2, exiting the pump, is then found using the expression for isentropic pumping of a liquid.

\[
h_2 := h_1 + v_1 \cdot (P_{\text{boiler}} - P_{\text{cond}}) \\
h_2 &= 97.031 \text{ BTU/lb}
\]

The enthalpy and entropy at State 3, exiting the boiler, are then found using the boiler pressure and maximum cycle temperature.

\[
\begin{align*}
h_3 &= h_{\text{T, H}_2\text{O}(T_{\text{bmax}}, P_{\text{boiler}})} \\
h_3 &= 1389.833 \text{ BTU/lb}
\end{align*}
\]

\[
\begin{align*}
s_3 &= s_{\text{T, H}_2\text{O}(T_{\text{bmax}}, P_{\text{boiler}})} \\
s_3 &= 1.568 \text{ BTU/lb-R}
\end{align*}
\]
Appendix - Rankine-Reheat Power Cycle Optimization (continued...)

The enthalpy at State 4, exiting the first turbine, depends on the reheat pressure and the isentropic efficiency of the turbine. Since no turbine efficiency is stated, a value of 100% is assumed. A vector of enthalpies at State 4 is evaluated based on each reheat pressure and the entropy at State 3.

\[ h_{4i} := h_{Ps\,H2O}(P_{rh_i}, s_3) \]

The enthalpy and entropy at State 5, exiting the reheat boiler, is determined for each reheat pressure and using the maximum reheat temperature.

\[ h_{5i} := h_{TP\,H2O}(T_{bmax}, P_{rh_i}) \quad s_{5i} := s_{TP\,H2O}(T_{bmax}, P_{rh_i}) \]

Like the first turbine, the second turbine is also assumed to be isentropic. State 6, exiting the second turbine, is then evaluated using the condenser pressure and the entropy at State 5.

\[ h_{6i} := h_{Ps\,H2O}(P_{cond}, s_{5i}) \]

With all of the states evaluated, the thermal efficiency is evaluated as the net work of the cycle divided by the net heat input into the cycle. The net work is the sum of the work of the first turbine, the second turbine and the pump. The net heat input is the sum of the heat delivered to the water in the main boiler and the reheat boiler.

\[ \eta_{1i} := \frac{(h_{5i} - h_{6i}) + (h_{3i} - h_{4i}) + (h_{1i} - h_{2i})}{(h_{3i} - h_{2i}) + (h_{5i} - h_{4i})} \]

The array of efficiencies is then plotted versus the reheat boiler pressure to identify the maximum thermal efficiency.

The maximum thermal efficiency of the reheat cycle is evaluated using the "max" function in MathCAD and is found to be 38% at a reheat boiler pressure of 309 psi.

\[ \max(\eta_{1i}) = 38.002\% \]