AC 2007-1004: USING A HOMEMADE LOW SPEED WIND TUNNEL TO ILLUSTRATE THE CONTINUITY EQUATION

Robert Edwards, Pennsylvania State University-Erie

Using a Homemade Low Speed Wind Tunnel to Illustrate the Continuity Equation

Robert Edwards
Penn State Erie, The Behrend College

Abstract:

The continuity equation is one of the most basic formulas used in fluid dynamics. The conservation of mass principle is crucial to most fluid flow problems, and it is that principle that is represented by the continuity equation. In a first course in fluid mechanics the continuity equation is usually applied by using average values for the velocity across any cross-section in the duct. However, it is important for the students to recognize that the equation applies for non-uniform velocity profiles also. This paper describes a lab experiment that can be used to illustrate the application of the continuity equation to a duct with a non-uniform velocity profile. The students take a grid of velocity data across two different cross-sections within the duct, calculate the total mass flow rate at each cross-section, and compare the two results.

I. Introduction:

The continuity equation states that for steady state operation, the mass flow rate at any cross-section in a duct is a constant. In an introductory course in fluid mechanics it is common practice to use average flow rate values across a cross-section when doing continuity equation calculations. It is very helpful to the students, however, to recognize that the continuity equation holds for velocity profiles other than uniform velocity. This concept can be presented in a lecture, and calculations can be performed to demonstrate the theory. Equation 1 gives the general equation for the mass flow rate at any cross-section.

$$\boxed{\dot{m} = \int_A \rho V dA} \quad \text{(Equation 1)}$$

Where:

ρ is density
V is velocity
A is area.

If the density is assumed to be constant, which is the case for most problems encountered in a first course in fluid mechanics then it can be brought outside the integral giving equation 2.

$$\boxed{\dot{m} = \rho \int_A V dA} \quad \text{(Equation 2)}$$

If the function for the velocity profile V as a function of position is known, then it can be evaluated at each cross-section.

In most real applications the velocity profile is not a nice clean function that can be easily integrated. The velocity can vary seemingly randomly across the cross-section. An example of this type of flow exists in the homemade wind tunnel shown in Figure 1 which has no flow straightening section at the inlet. This causes the velocity profile to be quite non-uniform at any given cross-section. Furthermore, the flow profile changes as the air moves through the tunnel.

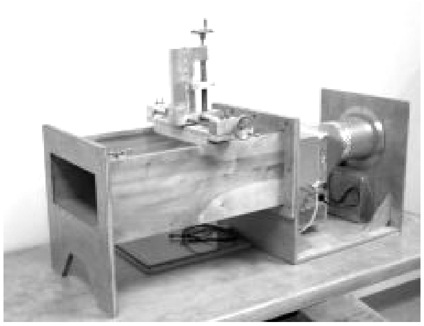

Figure 1

To determine the total mass flow rate at any given cross-section it becomes necessary to determine the mass flow rate for small areas within the cross-section and then find the sum of these small mass flow rates. Equation 1 then becomes equation 3. Again, if the density is constant it can be brought in front of the summation sign.

$$\boxed{\dot{m}_i = \sum_{i=1}^{n} \rho_i V_i A_i} \text{ (Equation 3)}$$

It is important for the students to recognize that the continuity equation holds for non-uniform velocity profiles such as those that exist inside this wind tunnel. This paper describes a lab experiment used for this purpose.

II. Equipment:

The following equipment is needed to conduct this lab:

- Homemade low speed wind tunnel (Figure 1)
- Hot wire anemometer
- Positioning mechanism for mounting the anemometer
- Means for determining the barometric pressure and the room temperature.

The wind tunnel was designed and built as a senior project by Mechanical Engineering Technology students. The wind tunnel has a test section that is 12 inches wide by 6 inches high. The maximum air velocity is approximately 2.5 m/sec. The test section and overall structure are constructed from plywood. A galvanized sheet metal section was fabricated to reduce from the 12"x 6" test section down to the fan diameter. Air is pulled through the wind tunnel by a variable speed axial fan.

The positioning mechanism for the hot wire anemometer, which can be seen mounted to the top of the wind tunnel in Figure 1, was specifically designed to be reversible so that the velocity profile could be obtained at both the inlet and the outlet of the test section. The horizontal and vertical position of the hot wire anemometer can be set using this positioning device. Vertically, the probe can reach within .5" of the floor of the section, and can theoretically reach the top wall. Unfortunately it can only reach within 1.25" of the side walls.

III. Test Procedure:

The strategy for determining the total mass flow rate at a cross-section of the wind tunnel in this experiment is to divide the total area into smaller areas, determine the mass flow rate for each of these smaller areas, and then sum all of them to find the total. The velocity is measured for each small area using the hotwire anemometer. The measured velocity is assumed to be constant across the small area. The test can be conducted using a constant air density, assuming that the air temperature is at room temperature, or using a variable density by measuring the temperature at each small area. The hotwire anemometer has a built in thermocouple which can be used to measure these temperatures. The mass flow rate is then determined for each small area, and all of the mass flow rates are summed to determine the total mass flow rate for the cross-section (see "Calculations" below). The smaller the test areas the more accurate the overall mass flow rate will be. However, there is lot of data to collect for this experiment so the test area needs to be reasonably sized to allow the students enough time for the test.

Velocity data is taken for an array of points across the cross-section. The grid is mainly comprised of 0.5" squares, however some adjustments have to be made along the side wall since the probe cannot reach to within 0.5" of the wall. Figure 2 shows the grid that is used. Data is taken at the intersections of the vertical and horizontal lines in that figure. Most of these data points are at the center of each small area, however adjustments need to be made for the sides and corners due to the limitations on travel of the positioning device.

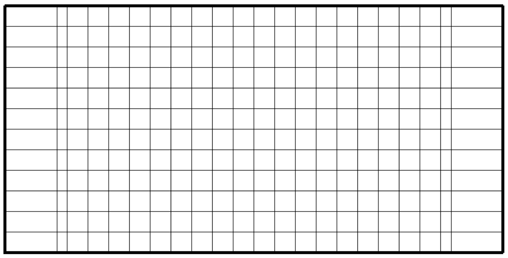

Figure 2

Before coming to lab the students are to prepare a data sheet for recording all of the data. The data that is required includes:

- Ambient temperature
- Barometric pressure
- Inside dimensions of the wind tunnel test section
- Velocity data at all of the data points shown in Figure 2. There must be enough space on the data sheet to record this data for two different cross-sections.

- Optionally, temperature data at all of the data points.

The hot wire anemometer is moved to each of the data points with the positioning mechanism, and the measurements are recorded. After all of the data is measured for one cross-section, the positioning mechanism is moved to the new cross-section and measurements are repeated. Since there are so many data points the task of data collection can become tedious, so lab sections are permitted to share data. That way each lab section only needs to collect data for one cross section. This does introduce some extra error into the experiment, but the goal is to help the students understand the concept, not necessarily to obtain extremely accurate results.

IV. Calculations

Equation 1 gives the general equation for mass flow rate at a cross-section. For this experiment the total mass flow rate is calculated as the sum of the individual mass flow rates represented by each data point. Equation 4 gives the expression for the total mass flow rate.

$$\boxed{\dot{m} = \sum_{i=1}^{n} \dot{m}_i} \quad \text{(Equation 4)}$$

The individual mass flow rate for area i is calculated using equation 5.

$$\boxed{\dot{m}_i = \rho_i V_i A_i} \quad \text{(Equation 5)}$$

The students are required to determine the density ρ_i based on the recorded room temperature. It is assumed to be constant for all of the areas. Optionally the temperatures at each data point can be measured and the density can be determined for each area. The velocity V_i associated with each area is the velocity measured at a grid point within that area. The area A_i associated with each velocity is determined by using the area formed by intersecting lines drawn half way between each data point. Areas along the edges extend to the wall of the wind tunnel. Figure 3 shows the locations of data points and area boundaries for the upper left corner of the wind tunnel. This type of pattern continues across the cross-section. Notice that the areas along the walls are different sizes than in the interior sections.

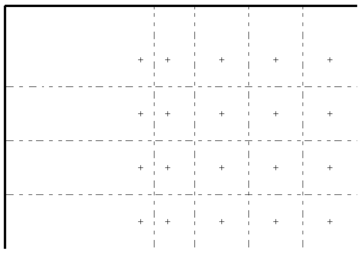

Figure 3

Table 1 gives the size of the individual areas. Dimensions are in inches for X and Y and in^2 for the area.

Location	**X**	**Y**	**Area**
Along side walls	1.375	.5	.688
Corners	1.375	.75	1.031
Along top and bottom	.5	.75	.375
Internal areas	.5	.5	.250

Table 1

After all of the data is collected it is compiled on a spreadsheet. All of the calculations required to determine the total mass flow rate for each of the two cross-sections is done on the spreadsheet. The results are then compared.

In order to help the students visualize the velocity profile across each cross-section a three-dimensional velocity plot is made using MATLAB or any other similar program. Figure 4 shows a typical example of this type of plot.

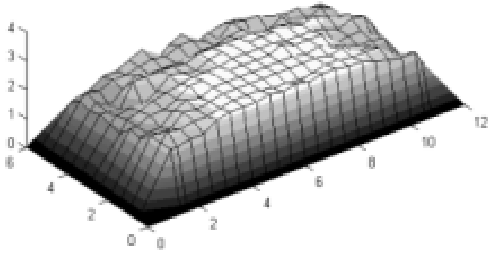

Figure 4

To create this plot the velocities at the walls are assumed to be zero. More information is given below on how to create this type of plot in MATLAB. Other similar programs could be used.

VI. Data and Results:

Table 2 shows a typical set of student data taken for this lab. The data is arranged to represent the locations of the data points within the cross-section when looking into the entrance to the wind tunnel. This is the data used for determining the overall mass flow rate at each cross-section. In order to create the three-dimensional velocity profile plots extra data is added along the walls. Velocity at the walls is assumed to be zero.

Notice that the velocity in the interior of the wind tunnel is fairly regular. However, there are large variations near the walls. This means that the wind tunnel is useful for lab experiments if the test specimens are placed near the center of the test section cross-section.

Upstream Cross-Section (Velocities in m/sec)

0.4	0.7	1.2	1.8	1.4	1.5	1.7	1.4	1.4	1.5	2.0	1.0	1.4	2.0	2.2	2.5	2.5	2.0	1.7	0.7	0.5
0.4	2.3	2.1	2.6	2.4	2.4	2.3	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.5	2.6	2.5	2.3	0.8	0.7	0.7
1.4	2.1	2.2	2.5	2.5	2.5	2.5	2.5	2.5	2.3	2.4	2.5	2.5	2.5	2.5	2.5	2.6	2.7	2.6	2.0	1.5
2.0	2.5	2.5	2.5	2.4	2.5	2.6	2.5	2.5	2.5	2.5	2.5	2.6	2.5	2.7	2.7	2.7	2.5	2.3	1.3	2.3
2.3	2.5	2.5	2.6	2.5	2.5	2.6	2.6	2.5	2.6	2.5	2.6	2.7	2.6	2.6	2.6	2.7	2.7	2.7	2.6	2.6
2.4	2.6	2.5	2.5	2.6	2.5	2.5	2.5	2.5	2.5	2.6	2.6	2.5	2.6	2.6	2.7	2.6	2.6	2.5	2.5	2.5
2.4	2.5	2.6	2.6	2.6	2.6	2.6	2.5	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.5	2.5
2.5	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.5	2.6	2.6	2.6	2.6	2.6	2.5	2.5	2.6	2.5	2.5	1.8	2.3
2.0	2.3	2.5	2.5	2.6	2.7	2.7	2.6	2.6	2.6	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.2	1.5
1.8	2.5	2.6	2.7	2.7	2.7	2.7	2.6	2.7	2.6	2.6	2.6	2.6	2.6	2.7	2.5	2.3	2.3	1.5	1.0	1.3
0.6	2.0	1.5	2.6	2.7	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.5	2.5	2.5	2.3	2.0	1.4	0.8	0.7

Downstream Cross-Section (Velocities in m/sec)

1.3	1.4	1.5	1.8	1.8	2.0	2.2	1.6	2.0	2.2	1.7	1.9	2.0	2.0	1.8	1.7	1.5	1.5	1.4	1.4	1.4
1.6	1.8	1.7	1.9	1.9	2.0	2.1	2.0	2.2	2.2	2.0	1.9	2.3	2.2	2.2	2.0	1.9	1.9	1.6	1.6	1.5
1.5	1.7	1.4	1.8	2.1	2.0	2.0	2.0	2.0	2.1	2.1	2.2	2.3	1.7	1.9	2.2	1.8	1.8	2.0	1.6	1.5
1.6	1.6	1.5	2.3	2.3	2.3	2.3	2.2	2.4	2.4	2.4	2.4	2.4	2.3	2.3	2.2	2.0	2.0	1.8	1.8	1.6
1.5	1.7	1.8	1.7	2.0	2.4	2.3	2.2	2.4	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.3	1.7	2.0	1.6
1.4	1.8	1.9	2.1	2.2	2.3	2.4	2.4	2.4	2.5	2.5	2.5	2.5	2.4	2.4	2.4	2.3	2.1	2.3	2.2	1.9
1.8	1.8	2.0	2.0	2.2	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.7	2.5	2.5	2.5	2.0	2.1	2.1	1.5	1.5
1.7	1.8	1.8	2.2	2.4	2.3	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.3	2.2	2.0	2.0	1.9	1.6
2.0	2.0	2.0	2.4	2.3	2.4	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.1	2.2	1.8	1.5	1.3
2.0	2.3	2.2	2.3	2.3	2.4	2.5	2.5	2.5	2.4	2.4	2.3	2.4	2.5	2.5	2.4	2.4	2.4	1.9	1.8	1.8
2.0	2.0	2.1	2.1	2.4	2.5	2.5	2.5	2.6	2.5	2.6	2.5	2.6	2.6	2.5	2.5	2.3	2.2	2.1	2.1	1.7

Table 2

When this particular set of data was taken the room temperature was 21^0 C. The density of air for that temperature at atmospheric pressure was looked up on a table in the students' textbook and found to be 1.2 kg/m^3. The calculated total mass flow rate at the upstream or entrance section to the wind tunnel is 0.124 kg/sec. At the downstream section the total mass flow rate is 0.116 kg/sec. These results are within approximately 6.2% of each other. This is typical of the type of error that is usually found in this experiment.

If the total mass flow rate is calculated using a single velocity value measured at the center of the wind tunnel it would be 0.145 kg/sec (assuming a measured velocity of 2.6 m/sec). This number is significantly higher than the results shown above.

VII. Typical Velocity Profiles:

In order to give the students a better picture of the velocity profile within the wind tunnel they are asked to create a short program in MATLAB to produce three-dimensional plots of the profile at each of the cross-sections. Any similar program could also be used. To do this they first need to create three data arrays:

x.dat:

This array gives the x-coordinates of the locations of the data points. Points along the left wall will have co-ordinates of x=0. The array will look something like:

```
0       1.25    1.5     2
0       1.25    1.5     2
0       1.25    1.5     2
```

etc.

An X coordinate is needed for each data point.

y.dat:

This array gives the y-coordinates of the locations of the data points. Points along the top wall will have co-ordinates of y=6. The array will look something like:

```
6       6       6       6
5.5     5.5     5.5     5.5
5       5       5       5
```

etc.

A Y coordinate is needed for each data point.

z.dat:

This array is for the measured velocity values. Values along each of the walls are all zero. All other values are measured data. The array will look something like:

```
0       0       0       0
0       .7      1.2     1.8
0       2.3     2.1     2.6
```

etc.

(Notice that zero is used for the velocity at the wall)

The Matlab program used to plot the 3-D velocity profile is:

```
% Plot the velocity field for a cross-section of the wind tunnel
load x.dat
load y.dat
load z.dat
contour3(x,y,z)
hold on
surf(x,y,z)
shading interp
```

This program loads the three data files, creates the contour, and produces a 3-D shaded color plot of the velocity profile. Figure 4 shows a typical plot of a velocity profile.

Other programs could be used to produce these plots. Figure 5 shows an example of a similar plot created as a surface plot in EXCEL.

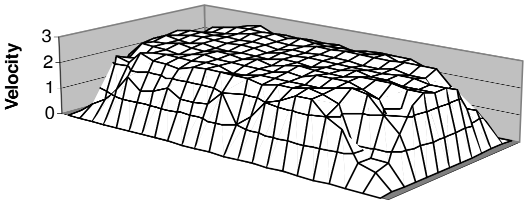

Figure 5

VII. Conclusions:

The continuity equation is a very important concept taught in all first courses in fluid and thermal sciences. Students use this equation regularly throughout any such course. This lab offers the students a unique opportunity to test the theory for a flow section with a non-uniform velocity profile.

ROBERT EDWARDS
Robert Edwards is currently a Lecturer in Engineering at The Pennsylvania State University at Erie where he teaches Statics, Dynamics, and Fluid and Thermal Science courses. He earned a BS degree in Mechanical Engineering from Rochester Institute of Technology and an MS degree in Mechanical Engineering from Gannon University.