AC 2007-1878: INTEGRATION OF ENGINEERING CONCEPTS IN FRESHMAN CALCULUS

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Integration of Engineering Concepts in Freshman Calculus

1. Introduction

Traditionally, basic sciences, physics and chemistry, and mathematics are required as core subjects for engineering education and have been taught independently by faculty members from mathematics and basic sciences. The National Science Foundation has awarded several projects to study mathematics and science education nationally. One of the awards is to the Center for Research on Education in Science, Mathematics, Engineering and Technology (CRESMET) at Arizona State University to investigate how best to support integrate instruction of mathematics, science, and engineering design. This investigation is an ongoing project at CRESMET and the disseminated results have shown the importance of integration of mathematical modeling in pedegogy and learning of scientific and engineering concepts. CASEE (The Center for the Advancement of Scholarship on Engineering Education, an operating center of the National Academy of Engineering) has coordinated several research projects on learning approaches in the engineering education[1]. Also, a NSF supported project on mathematical modeling in engineering education (M^2E^2) recently reported[2] the significance of mathematical simulation in engineering education and retention by testing in a freshman introductory course. In particular, the results give a positive observation on female engineering students. A recent review in January 2005 of the Journal of Engineering Education[3] has pointed out positive results of integrated engineering curricula in retention and diversity promotion and several future directions for research.

In this paper, we report our findings in running a pilot course for Calculus I (a required course for all students in the College of Engineering) with a new joint teaching approach by engineering and mathematics faculty members. The course contents have been developed by joint collaboration of the faculty members by the mathematics department and engineering faculty and it has been taught using team teaching. In this endeavor, the concept of integration of engineering concepts embedded in model-eliciting activities has been implemented. To examine the effect of the new approach, another group of freshman engineering students attended the same calculus course with a traditional teaching approach in the same semester. Assessment methodologies have been implemented to determine the degree of success on the students' learning and interest in the engineering profession.

2. Principles

In this section, we discuss the principles that we used when designing our engineering calculus sections.

2.1 *Illustration of engineering applications*. Many calculus textbooks include homework problems and examples that illustrate examples of mathematics. However, in most cases, these examples are mostly motivated by physics, with additional applications in economics and biology. Furthermore, many of these textbooks examples are somewhat artificial, contrived, and divorced from the real world. In our course redesign, we sought to expose our first-year students to relevant engineering problems in calculus.

2.2 *Contact with freshmen engineering students with engineering faculty*. In many engineering programs, it is common for first-year engineering students to take background courses in mathematics, physics, and other related subjects. An unfortunate consequence of this curriculum is that first-year engineering students often do not learn under the instruction of an engineering professor at this early stage of their academic careers. Our calculus sections were designed so that students would learn first-hand from an engineering professor about how concepts from calculus apply to engineering.

2.3 *Coverage of all mathematical topics covered in other calculus sections.* When designing our engineering calculus sections, we sought to ensure that students who enroll for these sections are exposed to the same mathematical ideas that students in the regular calculus sections see. We tried not to neither add mathematical ideas to the class nor take concepts out.

2.4 *Compatibility for transfer students and students with high school Advanced Placement credit*. While we hope to expose our students to engineering principles in the calculus sequence, we also realize that some engineering students will not take calculus at our university, either through transfer credit or the AP exam. While we want students taking our calculus sequence to receive specific exposure to engineering applications, we also do not want to construct a program where first-year students are expected to master advanced engineering principles in the calculus sequence, to the detriment of majors who did not take this sequence. For this reason, we chose not to introduce non-standard topics like vectors or force-body diagrams into our calculus class. That being said, we had no problems introducing the parlance of engineering in examples used to illustrate concepts from calculus.

2.5 *Flexibility for students who change majors*. A major goal of this redesign of our engineering calculus section is to retain first-year engineering majors. However, a fact of life is that some students change majors, either to or from engineering. Furthermore, at our university, high school students do not specifically apply for admission to the College of Engineering. Similar to our discussion above for transfer students or students with AP credit, we sought to create a program so that students who decide to become engineering majors after taking another calculus section will not be penalized for making this “late” decision. Furthermore, we have no desire to create a calculus sequence so specialized to engineering that they would be mathematically underprepared if they decide to switch majors.

3. Implementation

3.1 Contact with engineering faculty

A mathematics professor taught most of the engineering calculus course. However, time was built into the schedule for a faculty member from the College of

Engineering to make periodic 10- to 15-minute presentations about the applications of calculus course content to various advanced problems in engineering, including diffusion, permeability, and solubility. As would be expected, these presentations were presented from an engineering perspective as opposed to a mathematical perspective, including discussions of the apparatus used to physically measure these quantities. The intention of these presentations was so that first-year engineering students could learn first-hand about the types of problems faced by real engineers.

3.2 Creation of replacement homework problems

With the assistance of two local high school math teachers (Stephanie Guliano of Little Elm High School and Kathryn Day of Decatur High School), we read several engineering textbooks to identify and classify techniques from calculus that are necessary for mastery of these more advanced courses. We then sought to rewrite several of these problems in a context that first-year students would be able to understand, prior to their first class in either mechanical or electrical engineering. We tried to include examples from multiple branches of engineering to illustrate the importance of calculus for all of these fields.

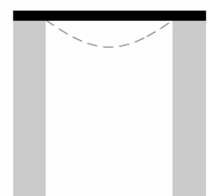
In doing this background reading, we noticed that, in examples presented by engineering textbooks, the final answer is often a formula instead of a numerical answer. This is in contrast to the approach favored by most calculus textbooks, which usually provide homework problems and examples where the final answer is a number. However, we see no obvious pedagogical reason for this limitation. We presume that this conceptual leap is just something that textbook publishers expect students to pick up on their own as they progress through the academic curriculum. Instead of simply trusting students to make this conceptual leap by themselves, however, we deliberately included many problems whose final answer is a formula instead of a number. We hope that writing these kinds of problems will help our students with their future engineering coursework.

In the sample problems that follow, there is often a fair amount of physics and/or engineering required to develop the claimed formulas from basic principles. However, as stated above, one overriding principle of our program is to not expect our students to perform advanced engineering computations that would not be reasonably expected of transfer students or other engineering students. Therefore, instead of expecting students to derive these formulas from their engineering context, the formulas are instead intended to expose students to engineering applications that will be more fully developed in their future courses. However, some reasonable amount of real-world context was provided in order to provide verisimilitude.

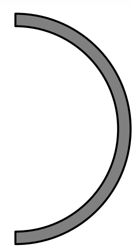
We also noticed it was straightforward to develop problems that required the use of either definite or indefinite integrals in engineering applications. However, a typical calculus course does not even introduce integration until halfway through the semester, and students are typically not expected to be comfortable with definite integrals until only two or three weeks remain in the course. For the first half of the class, finding realistic applications that only required knowledge of only derivatives and not integration was considerably more difficult. To impress upon first-year engineering students that calculus is actually relevant to their intended major, we were not content to simply defer significant engineering applications to the second half of the course. This provided a significant challenge to our project

Here we present some sample problems that we developed, in chronological and conceptual order of where they are presented in the calculus course. These problems were intended to replace other application problems that appear in our calculus textbook. As one would expect, the applications become more and more sophisticated as the course progresses.

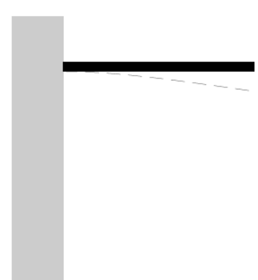
- The luminous intensity I (in candelas) of a lamp at voltage V is given by $I = (4 \text{ x } 10^{-4})V^2$. Determine the voltage at which the light is increasing at 0.6 candelas per volt.
- A structure consists of two horizontal bars of equal length L. The bars have pinned supports and are linked together in the middle. A vertical load P is applied at the junction, resulting in a complementary energy given by $U(P) = \dfrac{3P^{4/3}L}{4\sqrt[3]{EA}}$, where EA is the constant axial rigidity. The derivative dU/dP gives the displacement δ. Find δ. Your answer will involve P, L, E, and A.
- A simple two-meter beam is anchored at its endpoints. The beam supports a load uniformly applied across its entire length. Using principles from engineering, the deflection curve can be shown to be equal to $v(x) = x^4 - 4x^3 + 8x$. Find the absolute minimum and maximum values of $v(x)$ on the interval $0 \le x \le 2$.



- A shear force of magnitude V is applied to a semicircular cross section with thickness t and radius r. The shear stress is given by
$$\tau(\theta) = \frac{2V\sin\theta}{\pi rt},$$
where θ denotes the position along the semicircle. Find the maximum value of τ(θ) on the interval $0 \le \theta \le \pi$. Your answer will involve V, r, and t.



- The current in a circuit is given by $I = \int (3\sin t - 4\cos t)\, dt$. Find I.
- A cantilever beam of length L supports a uniform load of intensity q. The deflection $v(x)$ of the beam satisfies the conditions
$$v''(x) = \frac{qx^2}{2} - qLx + \frac{qL^2}{2},$$
$$v'(0) = 0,$$
$$v(0) = 0.$$
Solve this initial-value problem for $v(x)$. Your answer will involve q and L.



- A cantilever beam is subjected to a moment M_0 acting at the free end. The strain energy is defined to be equal to
$$U = \int_0^L \frac{M_0^2}{2EI}\, dx,$$

where the constant *EI* is the flexural rigidity of the beam. Calculate *U*. Your answer will involve M_0, *L*, and *EI*.

- The region between two concentric spheres of radii r_1 and r_2 is filled with conducting material with resistance ρ per unit volume. Both the inner and outer spheres are maintained at constant potentials, so there is a current radially outward from the inner sphere to the outer sphere through the conducting material. The cross-sectional area of the conductor at any radius r is given by $A = 4\pi r^2$, so that the resistance of the conductor is given by $R = \rho\int_{r_1}^{r_2}\frac{dr}{4\pi r^2}$. Find *R*.
- Neon is held in a cylinder fitted with a piston so that the gas can be slowly and frictionlessly compressed. The compression process is such that $p_i V_i^{1.3} = p_f V_f^{1.3}$, where p_i and p_f are the initial and final pressures, and V_i and V_f are the initial and final volumes. Determine the work input for this quasiequilibrium process by integrating $W = \int_{V_i}^{V_f}\frac{p_i V_i^{1.3}}{V^{1.3}}dV$.
- The endpoints of a beam of length *L* under a load tend to move closer together by a slight amount because of the deflection. This displacement, called the curvature shortening, is given by $\lambda = \frac{1}{2}\int_0^L [v'(x)]^2 dx$, where the deflection curve of the beam is given by $v(x) = \frac{4\delta}{L^2}\left(Lx - x^2\right)$. Compute λ.
- A disk of radius *R* has a uniform charge per unit area of σ. Calculate the electric field *E* along the axis of the disk at a distance *x* from its center by integrating $E = kx\pi\sigma\int_0^R \frac{2r\,dr}{\left(x^2 + r^2\right)^{3/2}}$.
- The power delivered to a loudspeaker is $P(t) = P_0 \sin^2(\omega t)$, where P_0 is the peak power and ω is a constant. Find the average power by computing the average of *P*(*t*) on the interval [0, π/ω].

4. Assessment

The same instructor (J. Liu) taught Calculus I in Fall 2005, Spring 2006, and Fall 2006. In Fall 2005, the course was taught in the traditional manner without the special presentations or the replacement problems. This class consisted of mostly non-engineering majors; a survey conducted at the end of the semester indicated 27 non-engineering majors and 4 engineering majors. In Spring 2006 and Fall 2006, the course was taught with the modifications discussed above. These classes consisted of almost exclusively engineering majors; an end-of-the-semester survey indicated a total of 24 engineering majors and 1 non-engineering major.

At the end of each semester, a survey was given to each class. The results of this survey are presented in the table below. Students were asked to answer on a 5-point scale, as follows: 1 = Not at All, 2 = To a Limited Extent, 3 = To a Moderate Extent, 4 = To a Great Extent, 5 = To a Very Great Extent.

Question	Avg 2005	Avg 2006
1. To what extent did you understand calculus before the start of this class?	2.33	2.33
2. To what extent do you understand calculus after completing this class?	3.63	3.54
3. To what extent did your instructor explain new concepts by making explicit links between what students already know and the new material?	3.50	3.46
4. To what extent were students required to be active participants in the teaching and learning process?	3.04	3.21
5. To what extent did the instructor/teaching methodology increase your understanding of the course content?	3.50	3.54
6. To what extent did classroom activities enhance your understanding and/or skills in the subject area?	3.21	3.29
7. To what extent have the learning experiences in this class increased your interest in the field of engineering?	1.96	2.71
8. To what extent do you think the content of this course is relevant to engineering?	3.92	3.63
9. To what extent do you find the course content relevant to your future studies?	3.25	3.42
10. To what extent did the instructor present real life applications of course content?	3.04	3.33
11. To what extent were homework assignments essential to the learning of the course content?	4.21	4.13
12. To what extent did course exams accurately assess your performance in this course?	3.79	3.88
13. To what extent were you taught how to apply knowledge and skills in new contexts?	3.33	3.42
14. To what extent was the textbook essential to the learning of the course content?	3.38	3.17
15. To what extent are you satisfied with the teaching/learning process in this course?	3.42	3.58

The only statistically significant difference in the responses occurred in Question 7 ($P = 0.026$). This consistency in the responses of the two cohorts is not too surprising given the modest sizes of the samples accumulated thus far. We would expect differences to become more and more statistically significant with further surveys. No other question produced a difference with an observed significance level less than 0.13.

At first blush, the inverted result of Question 8 is somewhat surprising, making it appear that the reforms described earlier have made calculus seem less relevant to engineering to first-year students. However, it is important to remember that the Fall 2005 class was taught to both non-engineering and engineering majors, who responded considerably different to this question (averages of 4.00 and 3.50, respectively). It is perfectly reasonable to expect that students majoring in biology or chemistry would have a higher expectation of the relevancy of calculus to engineering than the prospective engineers themselves.

Students under both modes of teaching calculus generally recognized that homework assignments were essential to the learning process, both on Question 11 and in their hand-written comments, making it difficult to isolate the effectiveness of the application problems that we developed. We also note that not one student volunteered in their hand-written comments that they found the presentations by engineering faculty to be either informative or instructive. This may be because no question was specifically asked about this component of the course reforms was not specifically assessed in the survey instrument.

5. Conclusions

The University of North Texas has begun to integrate engineering concepts into designated sections of the calculus sequence. The intention of these efforts is to improve retention of engineering majors and to illustrate engineering applications in these introductory courses. As these students progress through the academic curriculum, we will observe these two cohorts of students (regular vs. integrated calculus) to determine whether this alternative way of teaching calculus has measurable longitudinal effects for both retention as well as academic success.

Beyond the work presented in this paper, the Mathematics Department intends to continue coordination of efforts with the College of Engineering to design appropriate content that prepare and inspire prospective engineering students for their intended majors.

Acknowledgements
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References

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