

# **AC 2007-994: USING ENGINEERING MATHEMATICS TO LEARN STRUCTURAL ANALYSIS**

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# Using Engineering Mathematics to Learn Structural Analysis

## Abstract

Engineering students by the junior year are required to be proficient in mathematics. At this stage, the students have taken many of the introductory STEM (Science, Technology, Engineering, and Mathematics) courses. However, many students do not see nor appreciate the relevance of their mathematics courses to their major field of study. Beginning in structural analysis and in fluid mechanics in the junior year, the need for students to apply advanced mathematics becomes apparent. Students tend to want the calculator to do the math but many times fail to realize that calculators only generate numbers. Faculty must stand firm to motivate the students to learn and appreciate how to apply mathematical concepts to engineering problems and technology is helping to make this a reality.

This paper discusses how some of the fundamental content in a structural analysis course is presented and learned by the students. The content is presented in such a way that students must readily apply concepts learned in previous mathematics courses to be successful in the course. Students use technology such as a software package called Mathcad to help visualize the principles and applications that are discussed in the course.

## I. Introduction

Engineers are technical problem solvers. From a historical perspective of the mid 20<sup>th</sup> century and after, engineers have been trained to be number “crunchers” due to significant changes in engineering education and technology as a result of the post World War II era<sup>1-4</sup>. From high school math and science courses through college engineering courses, engineers have been “molded” to crunch numbers. Here is a problem with all the associated numerical information. Now, solve for the solution.

The practice of number crunching has not only been ingrained in our engineering youth but also in our technology. Computers and now calculators have been developed which can rapidly crunch numbers<sup>5</sup>. In terms of analyses, numerical based methods such as difference methods and finite element methods have been developed to approximate differential equations. Such solutions, even if the exact differential equations are known, generate only an approximate solution. And in the case of finite element analyses, the solutions are not conservative.

In engineering practice, number crunching has become routine. However, solutions are generated and constantly modified to meet unforeseen changes. After the solution has been calculated, modifying it is often done at considerable time and expense depending on the complexity of the problem and the dependency of the variable to other related system variables. It would be beneficial to teach engineers to develop general solutions which can be more routinely modified due to changes in constraints of variables or boundary conditions. Such solution strategies can be developed by solving problems symbolically in terms of variables rather than numerically.

One of the advantages of this problem solving technique is that students have an opportunity to develop equations in a pure mathematical form based on variables, devoid of arithmetic until the final step is performed. This gives students an opportunity to concentrate on the basic mathematical relationships between engineering variables<sup>6</sup>.

## **II. Background**

Engineers have traditionally used mathematics to study the behavior of nature for the purposes of human benefit. Over the last century, calculators, computers, computer languages and software have been developed for the purposes of rapid number crunching. However, questions remain regarding the influence of student learning using this approach. What impact of learning number crunching does this have on young engineers? Must engineers rely on a computer or calculator?

As part of ABET accreditation and curriculum programs at most institutions, students are required to be proficient in the basic principles of engineering mathematics, specifically: algebra, geometry, trigonometry, probability, statistics, calculus, and differential equations. For most civil engineering jobs, a working knowledge of calculus and differential equations is seldom needed to perform the necessary daily functions, and many students often know this. Many students seem to perceive the use of calculus and differential equations in the classroom as theory even though the use may be for application. And perhaps more troubling is the issue that students want the computer to solve the problems for them.

While in concept having the computer solve the problem is fine, students must clearly understand how the computer is solving the problem and be able to prove that the computer is in fact correct. Before students learn how to use software for complex problems, they need to understand, appreciate, and apply the fundamental principles of engineering. To do this, students must first be proficient in engineering mathematics, and faculty need to create and promote a learning environment that promotes this. It is easy for faculty to comment about the apathy of today's students and that, "When I was in school, things were different." Society does change over time, but as faculty it is our job to adapt and bring out the very best in our students, and technology used correctly can help.

In an effort to get students to think in terms of engineering mathematics in a more general sense rather than a number crunching approach, problems were solved in a junior level structural analysis course with a minimal amount of mathematical arithmetic. Problems were solved in terms of variables rather than numerical values so that students would better conceptualize the material rather than thinking in terms of magnitude. Students were required to solve mathematical expressions symbolically by hand and by calculator or computer software such as Mathcad.

## **III. Course Integration**

Approximately 50 percent of all the problems solved in class and on homework, quizzes and tests were required to be solved in terms of variables. The remaining 50 percent were solved using the number crunching procedure, which is in general more familiar to the students. By

using a 50/50 approach, the author hoped that students would come to understand and appreciate the benefit of solving problems in terms of variables.

In terms of course content, the classical methods of structural analysis, which require advanced engineering mathematics, such as, calculus and differential equations, were divided into categories. Each category consisted of a fundamental method or theorem. These categories include the following:

- Integration method
- Moment area theorems
- Conjugate beam method
- Work/energy method
- Virtual work method
- Castigliano's theorems
- Slope deflection method
- Methods for nonprismatic members

Typically, the methods and theorems are used to determine stresses, deflections, rotations, moments, and shears in structural members. The classical methods require students to readily perform multiple integrations, derivatives, and partial derivatives to solve problems. Some of the theorems and methods with graphical notation and corresponding equations as applied to a simply supported beam with a distributed load are given in Table 1.

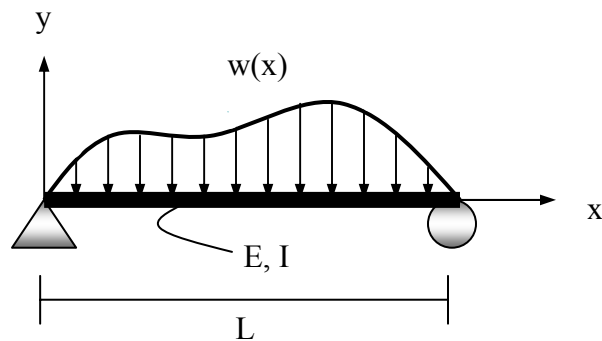
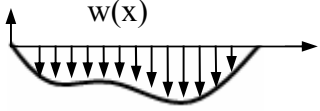
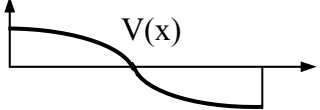
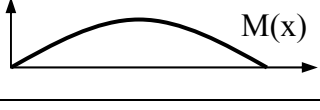
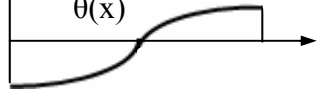



Figure 1. Simply Support Beam with a Distributed Load

Table 1. Classical Structural Analysis Methods

a) Integration Method

Component	Graphical Description	Governing Equations
Load, $w(x)$		$w(x) = EI \frac{d^4 y}{dx^4}$
Shear, $V(x)$		$V(x) = EI \frac{d^3 y}{dx^3} = \int w \, dx + C_1$
Moment, $M(x)$		$M(x) = EI \frac{d^2 y}{dx^2} = \iint w \, dx \, dx + C_1 x + C_2$
Slope angle, $\theta(x)$		$EI \theta(x) = EI \frac{dy}{dx} = \iiint w \, dx \, dx \, dx + \frac{C_1}{2} x^2 + C_2 x + C_3$
Deflection, $y(x)$		$EI y(x) = \iiint \int w \, dx \, dx \, dx \, dx + \frac{C_1}{6} x^3 + \frac{C_2}{2} x^2 + C_3 x + C_4$

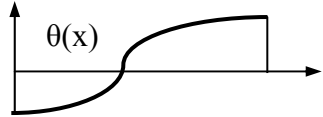
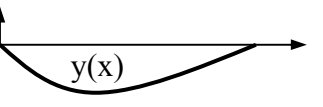
### b) Moment Area Theorems

Component	Graphical Description	Governing Equations	
Moment, $\frac{M(x)}{EI}$		$M(x)$ from statics due to $w(x)$	
Slope angle, $\theta_{a/b}$ Deviation, $t_{a/b}$		$\theta_{a/b} = \int_a^b \frac{M}{EI} dx$ $t_{a/b} = x_{a/b} \int_a^b \frac{M}{EI} dx$	where, $x_{a/b} = \frac{\int x_{a/b} dA}{\int dA_{a/b}}$

### c) Virtual Work Method

Component	Graphical Description	Governing Equations	
Moment, $M(x)$		$M(x)$ from statics due to $w(w)$	
Virtual unit moment couple, $P_\theta$		$P_\theta = 1$ at location of desired rotation	
Virtual unit load, $P$		$P = 1$ at location of desired deflection	
Virtual moment couple, $m_\theta(x)$		$m_\theta(x)$ from statics due to $P_\theta = 1$	
Virtual moment, $m(x)$		$m(x)$ from statics due to $P = 1$	
Slope angle, $\theta(x)$ due to $P_\theta = 1$		$\theta(x) = \int_0^L \frac{Mm_\theta}{EI} dx$	
Deflection, $y(x)$ due to $P = 1$		$y(x) = \int_0^L \frac{Mm}{EI} dx$	

#### d) Castigliano's Theorems

Component	Graphical Description	Governing Equations
Slope angle, $\theta(x)$		$\theta(x) = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial P_\theta} dx \quad P_\theta = 0$
Deflection, $y(x)$		$y(x) = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial P} dx \quad P = 0$

### III. Symbolic Problem Solving in the Classroom

All subjects throughout the semester were taught using the proposed methodology as well as the traditional method. One of the challenges in teaching the proposed methodology was the textbook, which was “Structural Analysis” by R. C. Hibbeler (2006). Even though the textbook is current and is an excellent source of reference, the material is presented in a more traditional and classical approach encouraging number crunching. Most examples are numerical in nature thus encouraging number crunching. In addition, some methods presented are no longer relevant such as the conjugate beam method. For example, to understand the derivation of the slope deflection equations as presented in the textbook requires the students to clearly understand the conjugate beam method, which is of little value to learn in the author’s opinion. Since the students were not taught this method in the course, the slope deflections equations were developed based on moment area theorems, which are more fundamental in the understanding of structural analysis than the conjugate beam method. Another shortcoming of the textbook was the lack of colored figures and diagrams. Nearly all of the figures, examples, and text were in black, white, or some shade of blue. Many textbooks in engineering are multicolored with colored photographs, which brings life to much of the content of the course. However, despite these pitfalls, the textbook is of significant value. Students can readily read the material and be able to apply the methodology to the problem at hand.

In general, the majority of engineering textbooks encourage number crunching as the primary methodology for problem solving. Read the problem, set-up the equations with the numerical values and solve. While this method and the proposed method may seem similar, they are fundamentally different. The use of variables in engineering textbooks is primarily limited to proofs of engineering equations. Most engineering students do not value the learning of proofs. They want to solve the problem not derive the fundamental expressions. Students tend to associate the use of variables as theory or proof work, which makes encouraging students to learn using variables a challenge.

Structural analysis textbooks do provide some example problems using variables such as the deflection at the free end of a cantilevered beam with a uniformly distributed load. However,

sample problems like this are limited. For the purpose of overcoming the challenge of the textbook and to demonstrate the practical significance of the proposed methodology, I explained the realities of engineering practice. Problems are seldom clearly defined. You may be given a problem and required to solve for the solution only to have to go back and re-work the problem many times often days, weeks or months after you have already done the calculation.

Having discussed the impact of the textbook on the course, the notes given in class played a significant role in how the material was first presented to the student and initially learned. Using the symbolic equations presented in Table 1, students were required to solve symbolically for deflection, rotation, moment, or shear in terms of variables  $w$ ,  $P$ ,  $L$ ,  $E$ ,  $I$ ,  $A$ , and  $x$  using a calculator or a computer software program such as Mathcad. Problems solved included: beams, trusses, and frames. The problems involved various loading conditions, boundary conditions, and changes in the cross-sectional area along the length of the beam. Different types of loading conditions included: point loads, uniformly distributed loads, non-uniformly distributed loads, and applied moments. Boundary conditions for beams ranged from cantilevered fixed ends to ends with rollers. For trusses, all connections were pinned or could be modeled as such. Also, nonprismatic beams were also examined, where the cross section area linearly changed across the length of the beam.

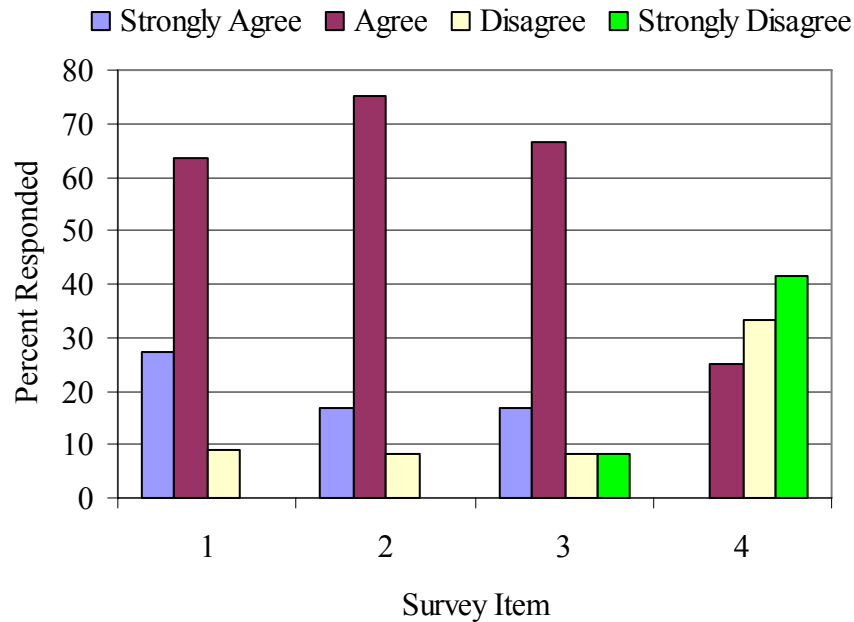
At the middle of the semester, the following question was posed to the students, "Is solving problems using variables teaching engineering theory or application?" Approximately, 64 percent of the class believed this to be theory, while the remaining believed this to be application. The majority of students incorrectly perceived engineering theory as working with variables even if the problem is application, such as solving for the deflection of a beam with a specified load. The students' perception of engineering application seems to be linked with dealing with physical numbers. Numbers have magnitude and, therefore, they have relative meaning. Variables do not have relative meaning unless they are compared in a specific context relative to each other.

## **V. Student Assessment**

Students were given a detailed survey at the end of the course. The students were asked, "How do you feel about solving problems in a more generalized manner in terms of variables? Did this feeling change over the course of the semester?" In general, the response was that students in the beginning did not appreciate solving problems this way, because the outcome was to develop a series of equations (rather than a specific numerical quantity). Dealing with numbers, the students felt more comfortable since they have relative meaning as opposed to variables, which have an unspecified magnitude. In addition, one student commented that when using variables, it is necessary to remember what all of the variables mean, which can be a daunting task for some types of the problems. However, the students (juniors) were more receptive in regards to using variables than students in a sophomore level engineering statics course that the author has previously taught using this methodology.

From statement 1 of Figure 2, 90.9 percent of the students either agreed or strongly agreed that they prefer to number crunch when solving problems. Using this approach, they tended to make mistakes resulting from this practice as indicated by statement 2 of Figure 2. According to

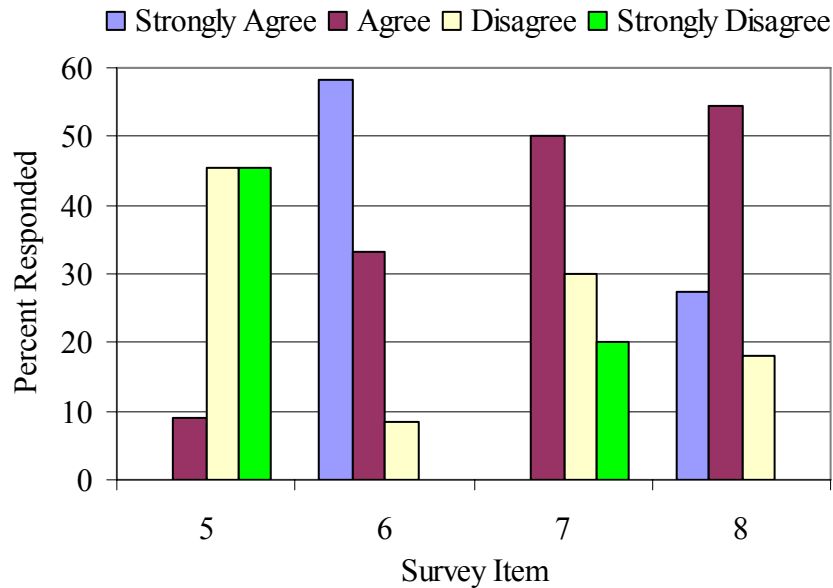
statement 3, 83.3 percent of the students either agreed or strongly agreed that they prefer using numerical based software such as spreadsheet programs. Statement 4 of Figure 2 indicates that 75.0 percent either disagreed or strongly disagreed that they prefer using more generalized mathematical software programs such as Mathcad or Maple.



1. I prefer to number crunch when solving problems. (Just give me the numbers, and I will solve the problem).
2. I make mistakes partially resulting from a number crunching approach to solving problems.
3. I prefer using numerical based software such as spreadsheet programs like MS Excel.
4. I prefer using more generalized mathematical based software such as Matchcad or Maple.

Figure 2. Student Survey – Part I.

Statement 6 of Figure 3, indicates that 83.3 percent of the students either disagreed or strongly disagreed that they prefer to learn about engineering theory. 91.6 percent of the students in statement 7 of Figure 3 indicated that they prefer learning about engineering applications. Both statements 6 and 7 demonstrate that engineering students are truly practical problems solvers as opposed to theorists. However, the students seem somewhat divided in terms of working with unspecified variables as indicated by statement 3 of Figure 2. According to that statement, 50 percent of the students agreed that they prefer to think in general terms such as “x” having an unspecified magnitude, while 50 percent either disagreed or strongly disagreed with this statement. However, statement 4 of Figure 2 seems to indicate the opposite, where 81.8 percent of the students either agreed or strongly agreed that working with numbers is less abstract to them than working with variables.



5. I prefer learning about engineering theory.
6. I prefer learning about engineering applications.
7. I prefer to think in general terms such that the distance “x” has some arbitrary magnitude.
8. I prefer to think in terms of numbers since the meaning is less abstract to me.

Figure 3. Student Survey – Part II.

Despite the initial reluctance of students to solve problems by developing generalized equations, students seemed to appreciate the methodology. However, this appreciation took time to develop, and careful presentation and preparation of the material content was required. Students found that one of the advantages to this methodology is that equations may be visually checked in a timely manner. Also, changes or modifications in the value of the variables may be performed more quickly than the traditional approach. It is the author’s opinion that students are not born thinking in terms of numbers but taught to think in terms of numbers. Greater emphasize needs to be place on the significance of variables as representing numbers and less on their quantitative magnitudes.

#### IV. Summary and Conclusions

In a junior level structural analysis engineering course, students were taught to solve problems by setting up and manipulating generalized equations using variables. The proposed methodology departs from the traditional approach, which emphasized number crunching techniques. The proposed methodology required students to solve problems for shears, moments, rotations, deflections, and stresses by solving engineering expressions symbolically in terms of variables. Afterward, the resulting expressions were manipulated and simplified. Finally, the numerical value was calculated at the very end by substituting all of the known variables. When the geometry of the problem was specified in terms of variables rather than

numbers, students often incorrectly perceived the problem to be engineering theory rather than application. Students often perceived engineering applications as working with numbers, and they perceived engineering theory as working with variables.

The course assessments indicated a preference by students to solve problems using the traditional approach despite being taught the proposed approach. While students over the course of the semester developed an appreciation for the proposed methodology, the students were still inclined to use the traditional approach. While the traditional approach does enforce learning, it does not in the author's opinion leave a lasting impression on the fundamental theories and concepts of applied structural engineering. Students and industry alike would be better served if engineering students were taught to think less in terms of numbers and arithmetic and more about the expressions that they are trying to solve.

### **Acknowledgements**

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